Math 3240
Topology 1, Assignment 3.

Due in class Thursday, February 27.

Questions from textbook:
Section 4.2: 16
Section 4.3: 2, 4, 11
Section 4.4: 7, 8, 12
Section 5.1: 9, 10, 13

Question A: Let

$$S^n = \left\{ (x_0, \ldots, x_n) : \sum_{i=0}^{n} x_i^2 = 1 \right\} \subset \mathbb{R}^{n+1}$$

equipped with the subspace topology. Prove that S^n is an n-manifold.

Question B:
Define an equivalence relation on S^n by declaring every point to be equivalent to its antipodal point, so the equivalence classes are $[\mathbf{x}] = \{\mathbf{x}, -\mathbf{x}\}$. Use the fact that S^n is an n-manifold to show that $S^n/\sim$ is an n-manifold as well. The manifold $S^n/\sim$ is commonly called $\mathbb{R}\mathrm{P}^n$. Note: For condition (iii) of a manifold, it is sufficient that you find an open neighbourhood of each $[\mathbf{x}] \in \mathbb{R}\mathrm{P}^n$ which is homeomorphic to an open subset of $\mathbb{R}^n$.