Math 3240
Topology 1, Assignment 2.

Due in class Tuesday, February 11.

Questions from textbook:
Section 3.4: 6, 9, 13
Section 3.5: 7, 11, 18
Section 4.1: 4, 7, 9
Section 4.2: 5, 6

Question A: Given three topological spaces X, Y and Z, topologize $X \times Y$ using the product topology, whose basis is all sets of the form $U \times V$ where U is open in X and V is open in Y. Show that $f : Z \to X$ and $g : Z \to Y$ are continuous functions if and only if

$$f \times g : Z \to X \times Y$$

defined by $(f \times g)(z) = (f(z), g(z))$ is continuous.

Question B:

(i) Let $p : X \to Y$ be a continuous map. Show that if there is a continuous map $f : Y \to X$ such that $p \circ f$ equals the identity on Y, then p is a quotient map.
(ii) If $A \subset X$, a **retraction** of X onto A is a continuous map $r : X \to A$ such that $r(a) = a$ for all $a \in A$. Show that a retraction is a quotient map.

Question C: Let $f : X \times Y \to Z$ be a map. We say that f is continuous in each variable separately if for each $y_0 \in Y$ the map $h : X \to Z$ defined by $h(x) = f(x, y_0)$ is continuous, and for each $x_0 \in X$ the map $g : Y \to Z$ defined by $g(y) = f(x_0, y)$ is continuous. Let $F : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by the equation

$$F(x, y) = \begin{cases} xy/(x^2 + y^2) & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

(i) Show that F is continuous in each variable separately.
(ii) Compute the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = F(x, x)$.
(iii) Show that F is not continuous.