

Math 3760
Advanced Calculus, Assignment 3.

Due January 2015.

Questions:

1. Evaluate the following Riemann-Stieltjes integrals, if they exist. Cite any theorems you use.

1. $\int_1^3 \alpha d\alpha$, where $\alpha : [1, 3] \rightarrow \mathbb{R}$ be $\alpha(x) = \lfloor x \rfloor x^2$.

2. $\int_0^3 gdf$ where $f(x) = \int_0^x e^{t^2} dt$ and $g(x) = e^{-x} \lfloor x \rfloor$.

2. Show that the Cantor set, with the classical “middle thirds” construction, is of measure zero. See Question 7.32 for more details, note I am not asking you to do that question.

3. Show that the function f which takes the value of 0 on rational numbers and 1 on irrational numbers is not Riemann integrable on $[0, 1]$.

4. Give (or look up) an example of a function with uncountably many discontinuities which is Riemann integrable. Show that the function has these properties.