Math 3322
Algebra 3, Assignment 6
Not for credit, but for practice.

1. Prove that the image of a module homomorphism is always a submodule.
2. An element m of an R-module M is called *torsion* if there exists a nonzero element $r \in R$ such that $rm = 0$. The set of all torsion elements is denoted $\mathrm{Tor}(M)$. Prove that if R is an integral domain then $\mathrm{Tor}(M)$ is a submodule of M, and give an example of a ring R and an R-module M for which $\mathrm{Tor}(M)$ is *not* a submodule.
3. Let $\phi : M \to N$ be an R-module homomorphism, where M and N are R-modules. Prove that $\phi(\mathrm{Tor}(M)) \subset \mathrm{Tor}(N)$.
4. Prove that if $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ and $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$ are not isomorphic as left $\mathbb{R}$-modules.
5. Prove that $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q}$ and $\mathbb{Q} \otimes_{\mathbb{Q}} \mathbb{Q}$ are isomorphic as left $\mathbb{Q}$-modules.