Math 3322
Algebra 3, Assignment 5
Due April 6 at the start of class.

1. Prove that if K is a subfield of $\mathbb{R}$ and if $f(x)$ is irreducible of degree 3 over K, then the discriminant D of $f(x)$ satisfies:
   - $D > 0$ if and only if f has three real roots, and
   - $D < 0$ if and only if f has precisely one real root.
2. Determine the Galois group of $x^3 - 2$ over $\mathbb{Q}$.
3. Determine whether or not $\mathbb{Q}\left(\sqrt{\frac{1+\sqrt{-3}}{2}}\right)$ is a Galois extension of $\mathbb{Q}$. If it is, calculate the Galois group.
4. Determine the Galois group of $x^4 - 5$ over $\mathbb{Q}$ and over $\mathbb{Q}(\sqrt{5})$.
5. Prove that if F is a radical extension field of K and E is an intermediate field, then F is a radical extension of E.
6. Prove that if F is a radical extension of E and E is a radical extension of K, then F is a radical extension of K.
7. **Bonus question:** Draw the lattice of subgroups of the Galois group of $x^4 - 5$ over $\mathbb{Q}$ and the corresponding lattice of fixed intermediate fields.