Math 3322

Algebra 3, Assignment 2

Due February 7 at the start of class (not February 5 as in the syllabus).

1. Prove that if G is a finite nilpotent group and m divides $|G|$, then G has a subgroup of order m. (Hint: To deal with groups of order p^n where p is prime, first deal with the case $n = 1$ and then proceed by induction on n to show that there are always subgroups of order p^k for $1 \leq k \leq n$. For the induction step, use the fact that $C(G)$ is nontrivial to find a normal subgroup of order p.)

2. Prove that every subgroup of a nilpotent group is nilpotent and every quotient of a nilpotent group is nilpotent.

3. Show that the permutation groups S_3 and S_4 are solvable but not nilpotent.

4. Using the previous two questions, conclude that the free groups $F(X)$ where $|X| > 1$ are not nilpotent.

5. Suppose that N is a simple normal subgroup of a group G and that G/N has a composition series. Then G has a composition series.

6. Prove that an abelian group has a composition series if and only if it is finite.