Lab Quiz 2.2 20 minutes

Name: Solutions. Student ID:

Always justify your answers!

Q1]... [4 points] Calculate the following limits.

(a)

$$\lim_{x\to-\infty} \frac{3x^3+x^2-1}{-x^3+2x-4} = \lim_{x\to-\infty} \frac{\frac{1}{x^3}(3x^3+x^2-1)}{\frac{1}{x^3}(-x^3+2x-4)}$$

1 pt for knowing this trick

$$= \lim_{x\to-\infty} \frac{3+\cancel{\frac{1}{x}}^{\to 0}-\cancel{\frac{1}{x^3}}^{\to 0}}{-1+\cancel{\frac{2}{x^2}}^{\to 0}-\cancel{\frac{4}{x^3}}^{\to 0}}$$

$$= \frac{3}{-1} = -3.$$

1 pt for getting the answer.

(b)

$$\lim_{x\to\infty} \frac{-e^x-6}{3e^x+3e^{-x}}-5 = \lim_{x\to\infty} \frac{\frac{1}{e^x}(-e^x-6)}{\frac{1}{e^x}(3e^x+3e^{-x})} + \lim_{x\to\infty}(-5)$$

1 pt for trick

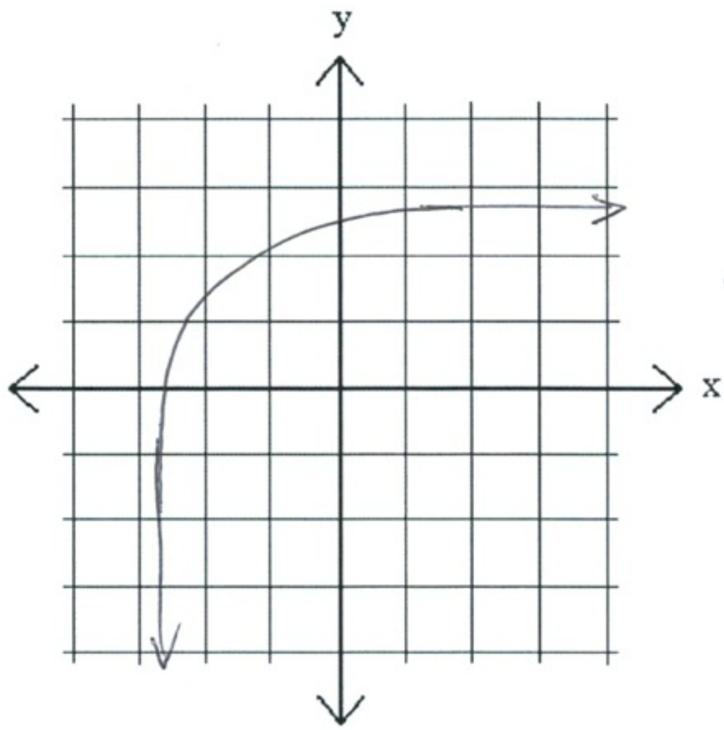
$$= \lim_{x\to\infty} \frac{-1-\cancel{\frac{6}{e^x}}^{\to 0}}{3+\cancel{3e^{-2x}}^{\to 0}} + (-5)$$

$$= -\frac{1}{3}-5 = -\frac{16}{3}$$

1 pt for corect answer

Q2]... [2 points] Sketch the graph of a function $f(x)$ satisfying

$$\lim_{x\to\infty} f(x) = 3 \text{ and } \lim_{x\to -3^+} f(x) = -\infty.$$



1pt for each limit correct.

Q3]... [4 points] Use derivative rules to calculate the equation of the line tangent to $f(x) = \sqrt{x} - x^2$ at $x = 1$.

$$f'(x) = \frac{1}{2}x^{-1/2} - 2x$$
$$= \frac{1}{2\sqrt{x}} - 2x.$$

So the slope is $\frac{1}{2(1)} - 2 = \frac{-3}{2}$.

2pts for slope. 1 pt for derivative, 1 pt for plugging in $x=1$.

The point on the line is $(1, f(1)) = (1, 0)$ since $f(1) = \sqrt{0} - 0^2 = 0$.

So $y = \frac{-3}{2}x + b$, passing through $(1,0)$ so

$0 = \frac{-3}{2}(1) + b$, so $b = \frac{3}{2}$.

$$\boxed{y = -\frac{3}{2}x + \frac{3}{2}}$$

2pts for correct value of b in $y = mx+b$.